Characterization of 3D fibrous media with geodesic methods

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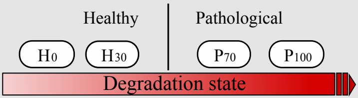
3-D Microstructure Meeting

(a) (b)

Motivations

- 3-D images of biological tissues
- 48 images classify in 2 or 4 classes : $\{Healthy, Pathological\}$, $\{H_0, H_{30}, P_{70}, P_{100}\}$:

Degradation of the tissues



Problematic

- Finding relevant features to classify healthy and pathological fibers
- Make a statistical analysis

Difficulties

- Anisotropy of the signal
- Small number of images $\Rightarrow$ overfitting
- Very high variability between tissues

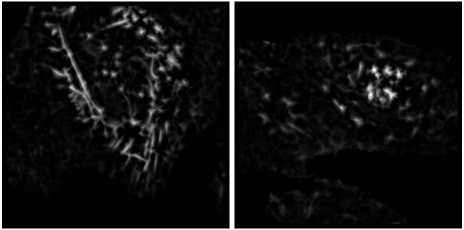
Pretreatments

Fibers enhancement

Linear *Difference of Gaussian* (DoG) to detect long structures
$\rightarrow$ Image of the fibers in grey scale

Segmentation of the fibers

No global threshold, then, we use an adaptive threshold for each slice of the 3-D image
$\rightarrow$ Mask of the fibers



(c) Healthy tissues H_0 (d) Pathological tissues P_{100}

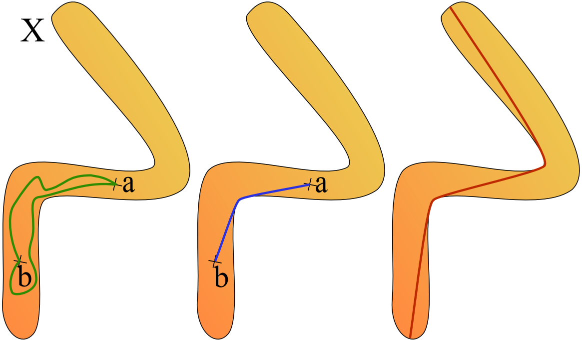
1. Introduction
   - Motivations
   - Pretreatments

2. Computation of the features
   - Geodesic methods
   - Skeleton methods

3. Statistical analysis
   - Correlation with the degradation
   - Statistical model

Geodesic diameter

Introduced by C. Lantuéjoul in [4].



(e) Arbitrary paths (f) Geodesic arc (g) Geodesic diameter : $L(X)$

Geodesy

With the barycentric diameter, we can define other attributes :

- Geodesic elongation

Definition (Geodesic elongation)

in 2D :

$$E(X) = \frac{\pi L^2(X)}{4S(X)}$$

in 3D :

$$E(X) = \frac{\pi L^3(X)}{6V(X)}$$

- Geodesic tortuosity

Definition (Geodesic tortuosity)

$$T(X) = \frac{L(X)}{L_{Eucl}(X)}$$

Geodesic attributes thinnings

The idea is to associate thinnings with geodesic attribute to get a new class of filters (Recently introduced in Morard et al. [5])

$$\psi_\chi(X_i) = \begin{cases} X_i & \text{if } \chi(X_i) \text{ is true} \\ \emptyset & \text{otherwise,} \end{cases} \tag{1}$$

Binary thinnings :

$$\rho_\chi(X) = \bigcup_i \psi_\chi(X_i). \tag{2}$$

Extension to grey scale images

$$\rho^{\mathrm{O}} f(x) = \vee\{h \in V \mid x \in \rho(X^h(f))\}. \tag{3}$$

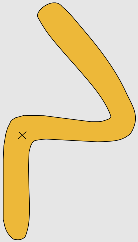
$$\rho^{\mathrm{T}} f(x) = \vee\{h \in V \mid \; x \in X_j^h : \exists\, \rho(X_i^k) \neq \emptyset, X_i^k \subseteq X_j^h, \forall k \geq h\}. \tag{4}$$

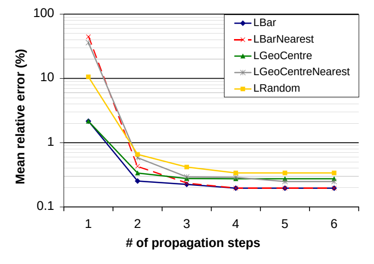
How to compute efficiently the geodesic diameter ?

Finding an approximation of the geodesic diameter

See Morard et al. [6] : *Efficient geodesic attribute thinnings*

Number of propagation ? Starting point ?

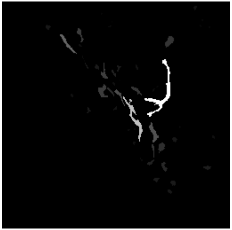




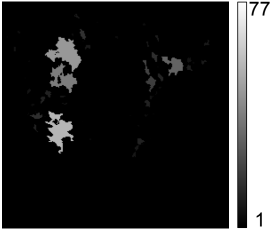
- The starting point is the farthest point from the barycenter of X
- Only 2 propagations

Barycentric diameter

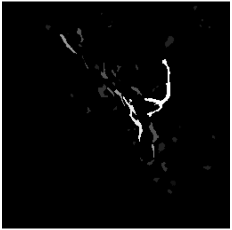
Geodesic methods



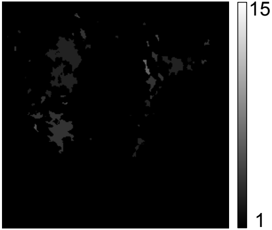
(h) Barycentric diameter (Healthy tissues S_0)



(i) Barycentric diameter (Pathological tissues P_{100})



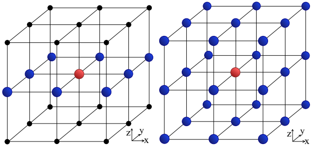
(j) Geodesic elongation (Healthy tissues S_0)



(k) Geodesic Elongation (Pathological tissues P_{100})

Features

We define 2 connectivities $C8$ et $C26$



(l) 2D treatment (m) 3D treatment

we get 6 features :

$$F_{L,C,\gamma} = \frac{Card\left(\rho_{L>\gamma,C}(f_{FB})\right)}{Card(f_{FB})} \times 100$$

$$F_{E,C,\gamma} = \frac{Card\left(\rho_{E>\gamma,C}(f_{FB})\right)}{Card(f_{FB})} \times 100$$

$$F_{T,C,\gamma} = \frac{Card\left(\psi_{T>\gamma,C}(f_{FB})\right)}{Card(f_{FB})} \times 100$$

Skeleton of the fibers, spatial features of the fibers

- Computation of the 3D skeleton [1]
- Pruning
- Suppression of the triple points.

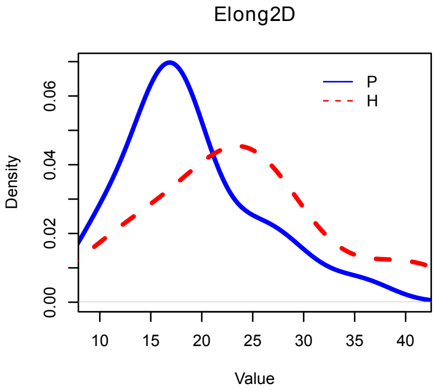
We get 3 features :

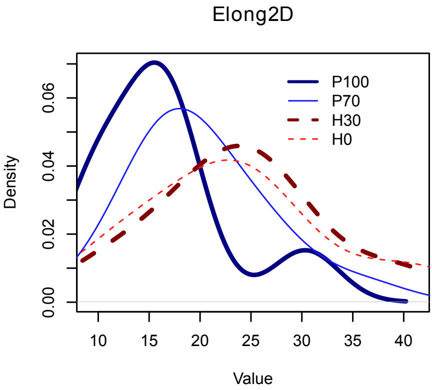
$$F_{SK_{C_{28}},NB} = CardCC(f_{Sk,C_{28}})$$

$$F_{SK_{C_{28}},MEAN} = \frac{\sum LengthCC(f_{Sk,C_{28}})}{F_{SK_{C_{28}},NB}}$$

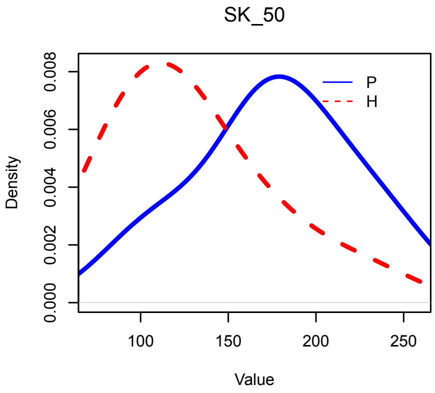
$$F_{SK_{C_{28}},\gamma} = CardCC(f_{Sk,C_{28}}) > \gamma$$

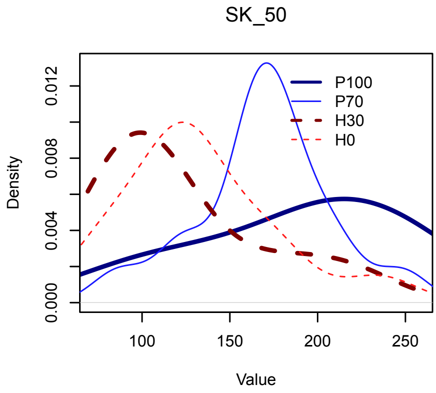
Correlation with the degradation





Correlation with the degradation





Statistical model

To avoid overfitting : $\rightarrow$ Linear regression

Feature selection :

- LARS, Efron et al. in [2],
- LASSO, Tibshirani in [7],
- Forward Stagewise Selection, Hastie et al. in [3].

Classification

TABLE: Prediction rate and the features are sorted by importance.

	OLS	Stagewise	LASSO	LAR
Prediction Rate	68%	75%	75%	73%
1	$F_{SK,MEAN}$	$F_{SK,MEAN}$	$F_{E,26}$	$F_{E,26}$
2	$F_{E,26}$	$F_{E,26}$	$F_{SK,MEAN}$	$F_{SK,MEAN}$
3	$F_{SK,NB}$	$F_{SK,50}$	$F_{L,26}$	$F_{SK,50}$
4	$F_{T,26}$	$F_{L,26}$	$F_{SK,50}$	$F_{T,8}$
5	$F_{SK,50}$	$F_{T,8}$	.	$F_{E,8}$
6	$F_{E,8}$	.	.	$F_{L,26}$
7	$F_{L,26}$	.	.	.
8	$F_{T,8}$	.	.	.
9	$F_{L,8}$	.	.	.

Validation method : *Leave One Out*

Conclusion

- Characterization of the fibers with geodesic methods
- A new attribute is introduce to speed up the computation : the barycentric diameter
- Characterization of the structures with the skeleton
- Statistical analysis with a *subset selection*

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A new thinning algorithm and its application to extraction of blood vessels, 1999.

B. Efron, T. Hastie, I. Johnstone, and R. Tibshirani.
Least angle regression.
Annals of statistics, 32(2) :407–451, 2004.

T. Hastie, R. Tibshirani, and J. Friedman.
The elements of statistical learning : data mining, inference, and prediction. volume2.
Springer Verlag, 2009.

C. Lantuejoul and S. Beucher.
Geodesic distance and image analysis.
pages 138–142, Salzburg, Australia, 1980. Mikroscopie.

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Geodesic attributes thinnings and thickenings.
In *International Symposium on Mathematical Morphology*. Springer, 2011.

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Efficient geodesic attribute thinnings (submitted).
Journal of Mathematical Images and Vision, 2011.

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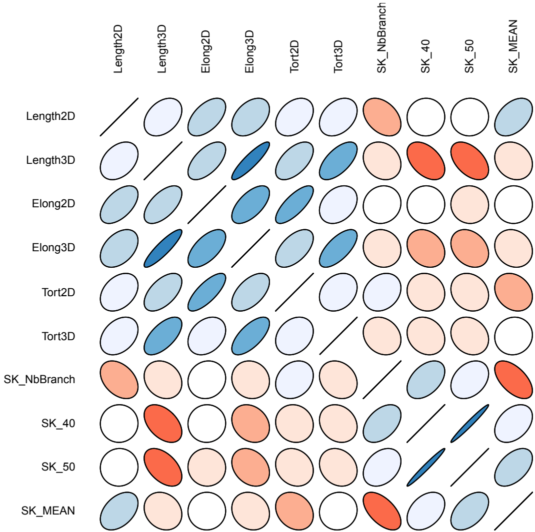
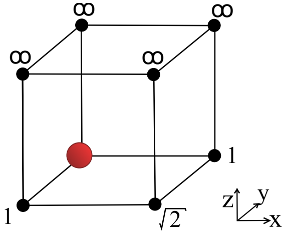
Regression shrinkage and selection via the lasso.
Journal of the Royal Statistical Society. Series B (Methodological), pages 267–288, 1996.

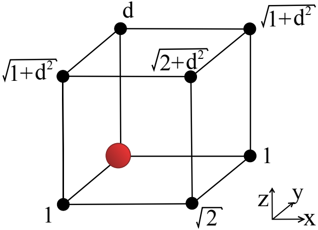
Annexe 1 : Correlation matrices



Annexe 2 : Anisotropy of the signal

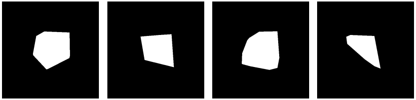


(n) C8



(o) C26

Annexe 3 : Binary objects



(a) Convex

(b) Pixel aggregation

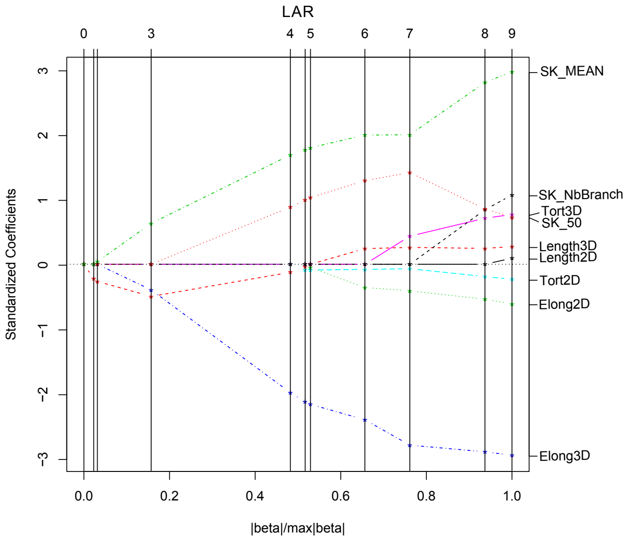
(e) Perlin Noise

(c) Ball aggregation

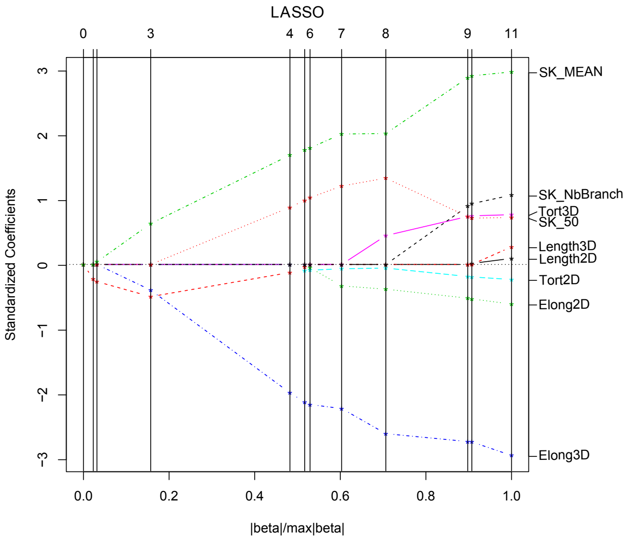
(d) Random walk

(e) Data base MPEG7

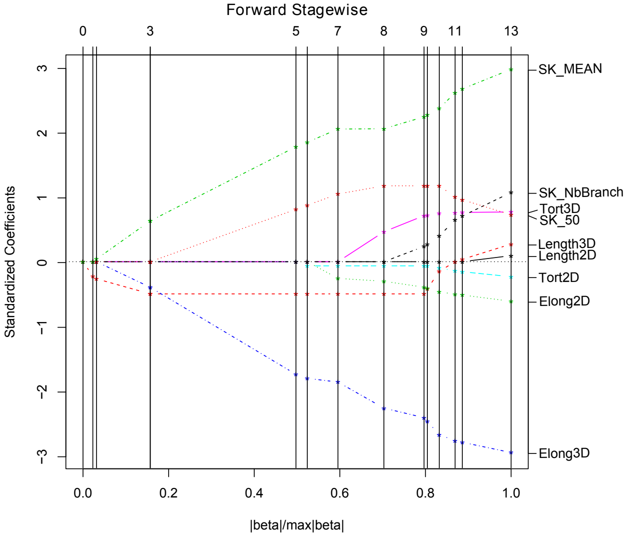
Annexe 4 : Regularization path LARS



Annexe 5 : Regularization path LASSO



Annexe 6 : Regularization path Forward stagewise selection



Annexe 7 : Correlation with the degradation